

Infrared radiation simulation of exhaust system by coupling FVM with narrow band k -distribution

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Abstract: A three-dimensional infrared radiation code for exhaust system was developed by the finite volume method coupled with narrow band k -distribution in non-gray absorbing-emitting media. The final infrared signature had considered the atmosphere effect, and the simulation values were favorably consistent with testing ones. The results indicate that the relative errors considering the effect of atmosphere compared with that of the contrary condition reduce by 31%, it shows that when simulating the infrared radiation of the target which is received by the infrared detectors, even the calculation band is in atmospheric windows 3~5 μm , the effect of atmospheric transmission on infrared signature of the target should not be neglected.

Key words: exhaust system; infrared radiation (IR); finite volume method (FVM);
narrow band k -distribution; atmospheric transmission

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With the deep development of infrared homing missile and infrared detectors, the extensive studies and researches of infrared radiation (IR) have been done in recent decades. The exhaust system, which includes the hot part after turbine, cavity and plume, is the one of the most important radiation sources. The research of the infrared characteristics of exhaust system becomes increasingly significant in the field of science and technology.

There are various relatively accurate methods developed to calculate gas radiation property parameters, such as line-by-line (LBL) method, statistical narrow band, wide band and k -distribution. The most accurate approach is the LBL^[1-2], however, it needs long calculation time and large computer resource. Thus, it is impractical to solve the engineering problems. In addition, wide band is a simplified model in which need low computational power but the precision

is low. Statistical narrow band models have higher accuracy, but are, due to the transmissivity approach, difficultly coupled with any desired solution method for radiative transfer equation (RTE)^[3]. Besides, narrow band k -distribution model which can avoid repeating the same calculation again and again by LBL approach has the greatly improved efficiency and is as accurate as LBL in the spectral integration computation^[4]. It has been widely used because of its high accuracy and availability in any desired solution method for radiative transfer equation.

A number of numerical simulation methods for solving the radiative transfer equation (RTE) have been developed, such as zonal method, discrete ordinate method (DOM), Monte Carlo method, finite volume method (FVM), etc. The zonal method was regarded as a basic approach to calculate the radiative heat transfer during the past decades. However, it is

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very hard to calculate the radiative heat transfer in nonhomogeneous media with anisotropic scattering^[5]. The conventional Monte Carlo method has been widely used to simulate radiative transfer problems because of its flexibility and simplicity in calculating the energy transport boundary conditions, but it takes long time to converge, and it has some difficulties to compatible with discrete ordinate method^[6] or finite volume method^[7-10]. The FVM was first proposed by Briggs L L et al^[10], in the field of neutron transport. It is derived for radiative heat transfer by Raithby G D, Chui E H et al^[11-13]. Slightly different schemes have been proposed by Chai J C et al^[14-16]. Up to the present, it has been successfully used to solve various problems of complex geometries^[17-19].

Since atmosphere is the medium between the detector and the target, the aircraft infrared signatures which are perceived by the detector must be affected by the intervening atmosphere. Thus, it is necessary to consider the influence of the atmosphere transmission on the target infrared characteristics Rao G A and Mahulikar S P^[20] described the importance of the atmosphere transmission for the research of IR characteristic modeling.

In this paper, the temperature, the pressure and the concentration distribution of exhaust system were calculated by numerical simulation of the 3-D (three-dimensional) flow field by employing FVM and RNG (renormalization group) $k-\epsilon$ turbulence model, which coupled with conduction, convection and radiation^[21]. FVM is also used to simulate the IR for coupling the radiation with the flow field which is evaluated by the same method. In the study, it is assumed that the engine operating in the non-afterburning state and the combustion of hydrocarbon fuel is complete. That is to say, the density of solid particles is rather low. Thus, the mixture of CO_2 and H_2O but no particles is taken into account when simulating the plume radiation. The gas radiative parameter was calculated by narrow band k -distributions. The attenuation of the ex-

haust infrared signatures due to atmosphere has been considered, and the homochromous atmospheric transmissivities under various conditions were obtained by LOWTRAN 7.

1 Mathematical formulation

1.1 RTE for narrow band k -distribution

The variations of gas absorption coefficient are very wild; the same value for k_η appears many times even across a very narrow spectrum, each time producing the identical intensity field within the medium. Consequently, carrying out LBL calculations across such a spectrum would be rather wasteful, repeating the same calculation again and again. It would, therefore, be advantageous to reorder the absorption coefficient field into a smooth, monotonously increasing function, assuring that each intensity field calculation is carried out only once.

The spectral RTE in the absorbing-emitting medium along a line of sight is expressed^[1]

$$\frac{dI_\eta}{ds} = k_\eta [I_{b\eta}(T) - I_\eta] \quad (1)$$

where I_η is the spectral intensity, $I_{b\eta}$ is the Planck function, k_η is the spectral absorption coefficient.

In the narrow band k -distribution model, Eq. (1) is multiplied by Dirac's delta function, $\delta(k - k_\eta)$ where, k is an absorption coefficient variable. Then, the RTE is integrated over a small spectral interval $\Delta\eta$, in which the Planck function is essentially constant. The RTE recorded in terms of the absorption coefficient variable k

$$\frac{dI_k}{ds} = k [f(k)I_{b\eta}(T) - I_k] \quad (2)$$

I_k can be defined as

$$I_k = \frac{1}{\Delta\eta} \int_{\Delta\eta} I_\eta \delta(k - k_\eta) d\eta = I_\eta f(k) \quad (3)$$

Where $f(k)$ is the k -distribution function for narrow band, and is evaluated as

$$f(k) = \frac{1}{\Delta\eta} \int_{\Delta\eta} \delta(k - k_\eta) d\eta = \frac{1}{\Delta\eta} \sum_i \left| \frac{d\eta_i}{dk_\eta} \right| \approx \frac{1}{\Delta\eta} \sum_i \left| \frac{d\eta_i}{dk_j} \right| [H(k_j + dk_j - k_{\eta,i}) - H(k_j - k_{\eta,i})] \quad (4)$$

$H(k)$ is the Heaviside's unit step function

$$H(x) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases}$$

Here the absorption coefficient is calculated by equation LBL with HITEMP database [22-23].

The RTE is more conveniently solved by expressing it in terms of the cumulative k -distribution function, which is written as

$$g(k) = \int_0^k f(k) dk \quad (5)$$

thus, the Eq. (2) now becomes

$$\frac{dI_g}{ds} = k_g (I_{b\eta}(T) - I_g) \quad (6)$$

where $I_g = I_k / f(k)$. Intensity averaged over the small spectral interval $\Delta\eta$ then follows as

$$\bar{I}_{\Delta\eta} = \frac{1}{\Delta\eta} \int_{\Delta\eta} I_{\eta} d\eta = \int_0^{\infty} I_k dk = \int_0^1 I_g dg \quad (7)$$

Eq. (7) can be calculated by Gauss quadrature scheme

$$\bar{I}_{\Delta\eta} = \int_0^1 I_g dg = \sum_{i=1}^N \omega_i I_{g,i} \quad (8)$$

where N is the number of quadrature points. The 7-point Gauss-Lobatto quadrature scheme is used in this paper.

Finally, the intensity of exhaust system in $3 \sim 5 \mu\text{m}$ ($2\,000 \sim 3\,330 \text{ cm}^{-1}$) can be obtained by summing over the narrow bands

$$I_{3 \sim 5 \mu\text{m}} = \sum_{l=1}^M \bar{I}_{\Delta\eta,l} \quad (9)$$

Where M is the number of narrow bands in $3 \sim 5 \mu\text{m}$.

1.2 RTE discretization by FVM

The exhaust system consists of the hot part after turbine, cavity and plume. The hot part after turbine and cavity was considered as an opaque diffuse solid surface. The radiation of hot plume was obtained by solving Eq. (6) using FVM.

Fig. 1(a) and Fig. 1(b) show the 3-D control volume of Cartesian coordinate grids and column coordinate grids, respectively. In figures, P indicates the element center point, e, w, s, n, t, b are the east, west, south, north, top, bottom face of the element, respectively, the corresponding uppercase is the element center

point in the same direction. Fig. 2 shows discrete of the zenithal angle or the azimuthal angle are uniform.

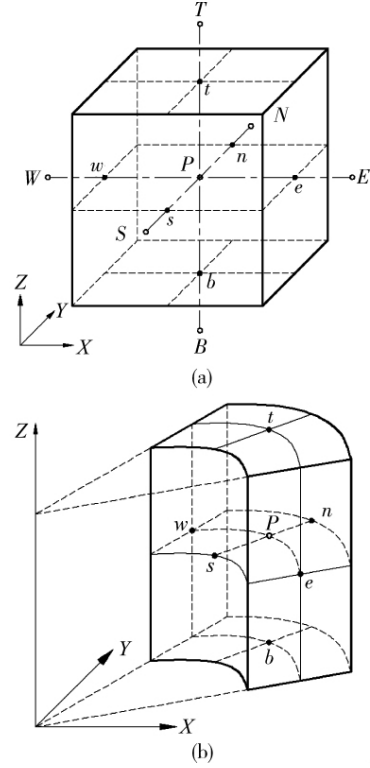


Fig. 1 Schematic of 3-D control volume

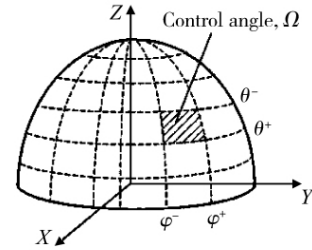


Fig. 2 Schematic of discrete control angle

Eq. (6) can be discretized by integrating over 3-D control volume V (Fig. 1), over a control angle Ω (Fig. 2), thus

$$\int_{\Omega} \int_{V_p} \frac{dI_g}{ds} dV d\Omega = \int_{\Omega} \int_{V_p} \kappa_g [I_{b\eta}(T) - I_g] dV d\Omega \quad (10)$$

From the divergence theorem [24], the left terms of Eq. (10) can be converted to a surface integral as follow

$$\int_{\Omega} \int_{V_p} \frac{dI_g}{ds} dV d\Omega = \int_{\Omega} \int_{A_c} I_g (s \cdot n_j) dA d\Omega \quad (11)$$

Thus, Eq. (10) leads to

$$\int_{\Omega} \int_{A_c} I_g(s \cdot n_j) dA d\Omega = \int_{\Omega} \int_{V_p} \kappa_g [I_{b\eta}(T) - I_g] dV d\Omega \quad (12)$$

In FVM, for the term on the left hand side, the intensity is assumed to be constant across each face of the control volume as well as the control angle. Similarly, it is assumed, for the term on the right hand side, that values are constant throughout and equal to the value at point P . Then, Eq. (10) can be written as

$$\sum_{j=1}^M A_j D_j I_{g,j} \approx \kappa_{g,p} [I_{b\eta} - I_g]_p V_p \Omega \quad (13)$$

where subscripts j and p imply evaluation at the center of the control volume's face A_j and control volume center P , respectively; D_j is the direction weight function which is defined as

$$D_j = \int_{\Omega} (s \cdot n_j) d\Omega \quad (14)$$

Since the stability considerations discussed by Chai J C et al^[25], the step scheme is generally preferred. Thus, this scheme was used in the paper. That is to say, when $D_j > 0$, $I_{g,j} = I_{g,p}$. The Eq. (14) then leads to the final expression

$$a_{g,p} I_{g,p} = \sum_{j=E,W,S,N,T,B}^M a_j I_{g,p} + b_{g,p} \quad (15)$$

Where

$$a_{g,p} = \sum_{j=e,w,s,n,t,b}^{M_c} \max[A_j D_j, 0] + \kappa_{g,p} V_p \Omega \quad (16)$$

$$a_j = \max[-A_j D_j, 0] \quad (17)$$

$$b_{g,p} = \kappa_{g,p} \bar{I}_{b\Delta\eta,p} V_p \Omega \quad (18)$$

For an opaque diffuse solid surface, the boundary condition can be written as

$$I_{g,w} = \epsilon_w \bar{I}_{b\Delta\eta,w} + \frac{1 - \epsilon_w}{\pi} \sum_{D_w^n < 0} I_{g,w}^n |D_w^n| \quad (19)$$

Where subscript w imply the wall surface, the superscript n indicate the n th control angle of the plume radiance at adjacent element.

1.3 Atmospheric transmissivity

There are a variety of gases in atmosphere, such as N_2 , O_2 , CO_2 , H_2O , O_3 , trace gases, etc., in addition, there are some particles called aerosol like dust, water droplet, etc. in atmosphere. The properties of these species depend on

the altitude and the latitude; hence, it is extremely difficult to calculate the atmosphere transmissivity.

The LOWTRAN 7, which was released by the Air Force Geophysical Laboratory, USA, is available for calculating atmospheric transmissivity in various conditions. The LOWTRAN 7 code is widely used in present analysis for evaluating atmospheric transmissivity. Fig. 3 shows that atmospheric transmissivity computed using the LOWTRAN 7 code for midlatitude summer for a vertical path length of 5 m and 5 km in the band 1~20 μm .

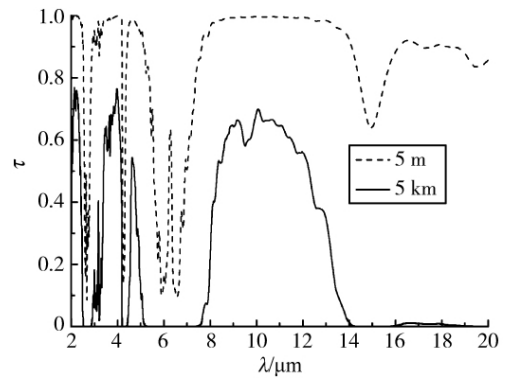


Fig. 3 Atmospheric transmissivity

As shown in Fig. 3 there are several bands in the total IR spectrum where atmospheric transmissivity is high. These bands usually known as atmospheric windows commonly include the band 3~5 μm and the band 8~14 μm . The IR detector must operate within these atmospheric windows. Atmospheric transmissivity is high and absorption is low in the atmospheric windows, the effect of atmosphere, however, still cannot be neglected. It is quite necessary to consider the effect of atmospheric attenuation when the IR signature of aircraft is calculated.

2 Results and discussions

One case is calculated by the method presented above to validate this work. The temperature, the pressure and the concentration distribution is obtained by a computational fluid dynamics (CFD) code which is worked out solving

the Navier-Stokes (N-S) equations using the FVM and RNG k - ϵ turbulence model. The diameter of nozzle outlet is 0.072 m (Fig. 4); the total number of spatial control volumes in this simulation is 909 312 computational domain. The total temperature, the total pressure and the Mach number of nozzle inlet are equal to 810 K, 11 000.0 Pa and 0.15, respectively, and the environmental pressure is 101 325.0 Pa.

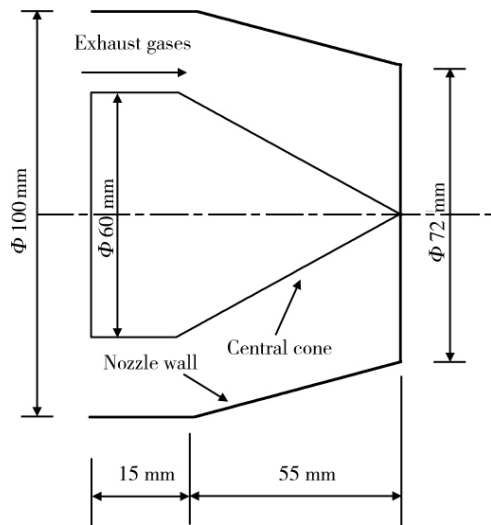


Fig. 4 Structure sketch of testing nozzle

The emissivity of the engine cavity and the entrance of the nozzle which is assumed as a solid surface is a constant ($\epsilon_w = 0.8$), the total number of spatial control volumes in the IR simulation is 14 208 (1/64 of the CFD number), the solid angle is divided into $(N_\theta \times N_\varphi) = (12 \times 18)$ control solid angles.

Fig. 5 (a) is the plot of nozzle mesh, the spatial domain is discretized into five blocks of grids. Fig. 5 (b) is a sectional view, and Fig. 5 (c) is the center part of side view. Fig. 6 shows the temperatures distribution and the molar ratios of CO_2 (which is similar with that of H_2O).

Fig. 7 presents the schematic of the detection angle (θ), namely the zenithal angle when the azimuthal angle is 0° , that is to say, the detector detects the IR radiation of the nozzle at horizontal level with different detection angle. The detection range is 5 m.

The comparison of calculating and experiment data is shown in Fig. 8. Where I_0 is experi-

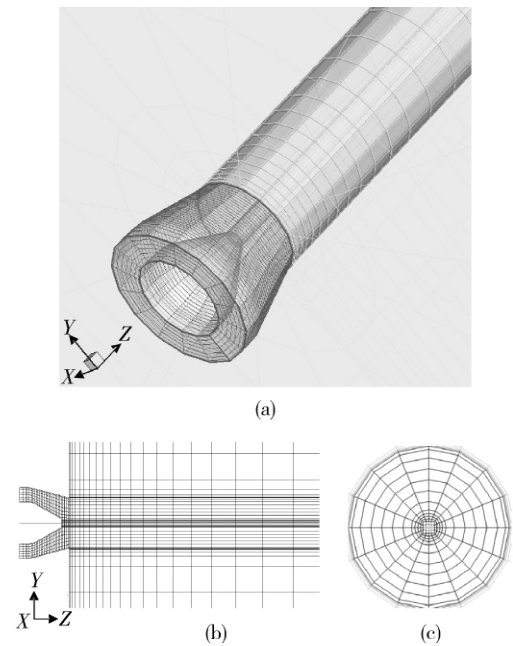


Fig. 5 Mesh of nozzle

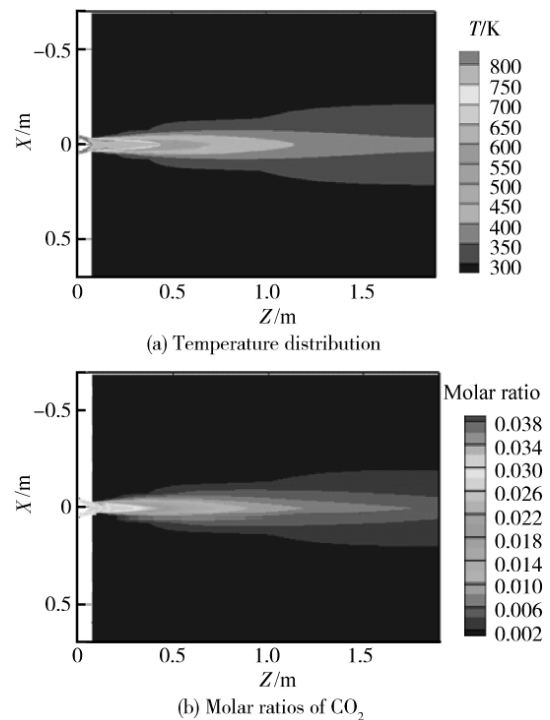


Fig. 6 Results of temperature field and concentration field calculated by CFD code

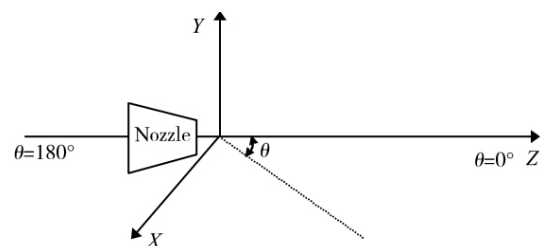


Fig. 7 Schematic of detection angle

ment result, I_1 and I_2 show the computational values without considering atmosphere effect and that with the atmosphere effect, respectively. The value of I_1 or I_2 is an integral of the spectral radiation over the $3\sim5\text{ }\mu\text{m}$ band. As shown in Fig. 8, the shape of cavity-plume infrared radiation is like a half ‘pear’, when the zenithal angle is increasingly close to 0° , the value increases quickly at first then decreases slowly. The IR radiation of the cavity and the central cone is the most critical factor for this phenomenon, because the cavity and the central cone emit radiation continuously, and it contributes most part of the total radiation.

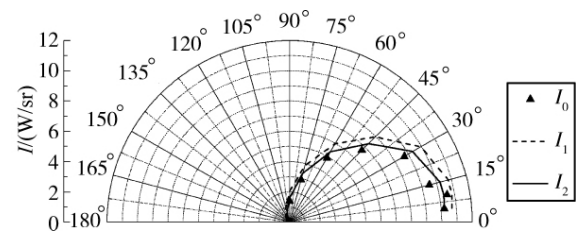


Fig. 8 Comparison of calculating and testing values

In addition, though the detection range is just 5 meters, it is obvious that the results which consider the effect of atmospheric attenuation are in better agreement with the testing values.

The maximum relative error of I_1 gets up to 46%, but the maximum error of I_2 is less than 15%.

Table 1 presents the relative errors between experimental and calculating values, E_1 and E_2 are the relative error of I_1 and I_2 , the expression of them is $E_{1/2}=(I_{1/2}-I_0)/I_0$, and ϵ_{12} is equal to E_1-E_2 . As Table 1 shows, with the increase of the detection angle, E_1 and E_2 become larger; it is based on the length of plume core area. The plume radiation share of total IR radiation becomes larger and oppositely the nozzle radiation share gets smaller with the increase of zenithal angle. The emissive band of exhaust plume is very narrow which is mainly in the $4.15\sim4.45\text{ }\mu\text{m}$ band and they agree well with atmospheric absorption bands. Consequently, most IR radiation emitted by exhaust plume is absorbed by the intervening atmosphere. Hence, the larger the zenithal angle is, the stronger the impact of atmospheric attenuation on simulation of IR signature gets, and i. e. the greater the relative error without atmospheric attenuation becomes (shown in Table 1). That implies that the role of atmosphere should not be neglected when simulating the IR signature of the exhaust system.

Table 1 Relative errors between experimental and calculating values

$\theta/(^\circ)$	$I_0/(\text{W}/\text{sr})$	$I_1/(\text{W}/\text{sr})$	$I_2/(\text{W}/\text{sr})$	$E_1/\%$	$E_2/\%$	$\epsilon_{12}/\%$
5	10.2	10.71150	10.21677	5.054	0.164	4.890
10	10.5	10.81476	10.28259	2.997	-2.071	5.068
15	9.5	10.79032	10.22674	13.582	7.650	5.932
30	8.7	9.95085	9.32919	14.378	7.232	7.146
45	6.7	7.96213	7.33662	18.837	9.502	9.335
60	4.9	5.97825	5.37671	22.005	9.729	12.276
75	2.9	3.87490	3.32497	33.617	14.654	18.963
90	1.4	2.03745	1.60089	45.532	14.349	31.183

The atmospheric transmissivities in the band $3\sim5\text{ }\mu\text{m}$ for the path length of 5 m and 5 km are shown in the Fig. 9. As the figure indicates, there is a narrow band of strong absorption $4.2\sim4.5\text{ }\mu\text{m}$ in the band $3\sim5\text{ }\mu\text{m}$ for the path length of 5 m, while the absorption in other

bands is relatively weak, and the transmissivities are close to 1. For the condition of 5 km, however, the transmissivities obviously descend, and two bands of strong absorption $3.0\sim3.15\text{ }\mu\text{m}$ and $3.15\sim3.45\text{ }\mu\text{m}$ are emerged, in addition, the absorption band $4.2\sim4.5\text{ }\mu\text{m}$ is broad-

ened to $4.0 \sim 4.65 \mu\text{m}$.

The results presented from Fig. 10 are calculated at the zenithal angle of 28.125° and the azimuth angle of 345° . I_{cp} indicates the spectral radiative intensity without the impact of atmosphere, I_{cpa1} and I_{cpa2} are the spectral radiative intensity with considering the path length of 5 m and 5 km, respectively.

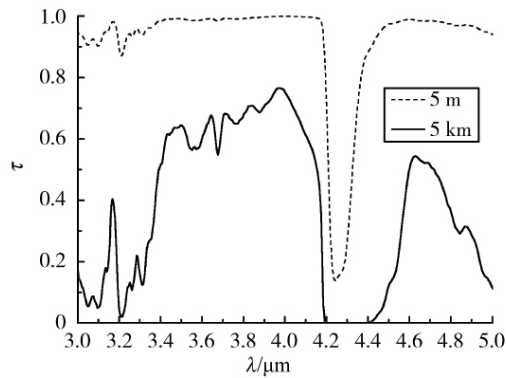
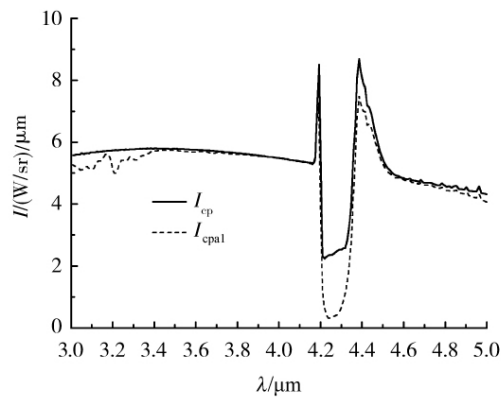
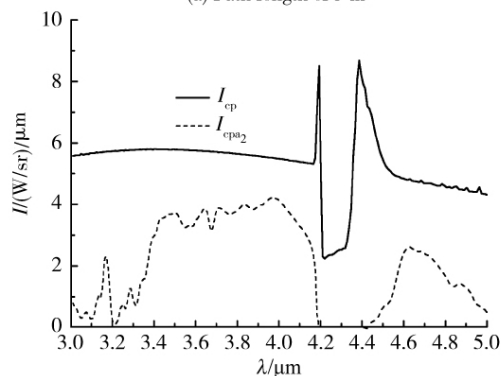


Fig. 9 Atmospheric transmissivity in $3 \sim 5 \mu\text{m}$



(a) Path length of 5 m



(b) Path length of 5 km

Fig. 10 Spectral radiative intensity in $3 \sim 5 \mu\text{m}$ band for different path length

As Fig. 10(a) shows, due to the effect of atmosphere, the spectral radiative intensity

sharply decreases in the band $4.2 \sim 4.5 \mu\text{m}$, it is responsible for the decline of integral value in the band $3 \sim 5 \mu\text{m}$. Besides, As shown in Fig. 10 (b), the spectral radiative intensity obviously descend in whole $3 \sim 5 \mu\text{m}$ band, especially in the band $3.0 \sim 3.15 \mu\text{m}$, $3.15 \sim 3.45 \mu\text{m}$ and $4.0 \sim 4.65 \mu\text{m}$.

3 Conclusions

A code for cavity-plume combination infrared radiation in 3-D enclosures based on FVM was developed. The temperature field, the pressure field and the concentration field of exhaust system were calculated by a computational fluid dynamics code, which coupled with conduction, convection and radiation. The gas radiative properties models, a general formulation of the finite volume method, and the atmospheric transmissivity were presented. The IR radiation of exhaust system is dominant in the band $3 \sim 5 \mu\text{m}$. The influence of atmosphere is often neglected because the band is in the atmosphere window. The results, however, turn out that the atmosphere still affects target IR signature even in the atmosphere window. Especially when the detection range is relatively far, the difference between the computed value and the measured value will become remarkable if the effect of atmosphere is overlooked.

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